

ÁLGEBRA

1. Resuelve:

$$\left. \begin{array}{l} \text{a. } \frac{x+1}{2} - \frac{y-2}{3} = \frac{4}{3} \\ \frac{2(x+y)}{3} - \frac{x+2y}{2} = \frac{1}{3} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{b. } x^2 + y^2 = 20 \\ x^2 - y^2 = -12 \end{array} \right\}$$

$$\left. \begin{array}{l} \text{c. } (x-y)^2 = 16 \\ 2x + 4y = 3(y-2) + 20 \end{array} \right\}$$

$$\text{d. } 4x^4 + 15x^2 - 4 = 0$$

$$\text{e. } 4x^4 - 64 = 0$$

2. Factoriza los siguientes polinomios y di cuáles son sus raíces:

$$\text{a. } 6x^5 - 7x^4 - 9x^3 + 7x^2 + 3x$$

$$\text{b. } x^5 - 9x^4$$

$$\text{c. } 49x^2 - 16$$

$$\text{d. } 9x^4 - x^2$$

$$\text{e. } 16x^2 - 8x + 1$$

$$\text{f. } 2x^4 + 12x^3 + 18x^2$$

$$\text{g. } x^5 + x^4 - 2x^3 - 2x^2$$

$$\text{h. } x^4 - x^3 - 3x^2 + 3x$$

3. Realiza las siguientes divisiones, y calcula cociente y resto:

$$\text{a. } (6x^4 + 8x^2 + 7x + 40) : (2x^2 - 4x + 5)$$

$$\text{b. } (x^5 - 7x^4 + x^3 - 8) : (x^2 - 3x + 1)$$

$$\text{c. } (6x^4 - 2x^2 + 4x - 2) : (x^2 - x + 1)$$

$$\text{d. } (x^5 - 3x^2 + 4x - 2) : (x^2 - 2x + 2)$$

4. Aplica la regla de Ruffini para dividir:

$$\text{a. } (5x^4 + 6x^2 - 11x + 13) : (x - 2)$$

$$\text{b. } (6x^5 - 3x^4 + 2x) : (x + 1)$$

$$\text{c. } (7x^2 - 5x^3 + 3x^4 - 2x + 13) : (x - 4)$$

$$\text{d. } (4x^3 - 9 - 51x^2 + 6x^4 - 3x) : (x + 3)$$

5. Utiliza la regla de Ruffini para hallar $P(a)$:

$$P(x) = 7x^4 - 5x^2 + 2x - 24, \quad a = 2, a = -5, a = 10$$

6. Simplifica las siguientes fracciones.

$$\text{a. } \frac{2x^2 - 6x}{4x^3 - 2x}$$

$$\text{b. } \frac{x^3 + 3x^2 + x + 3}{x^3 + 3x^2}$$

$$\text{c. } \frac{x^2 + xy}{x^2 + 2xy + y^2}$$

7. Opera y simplifica:

$$\text{a. } \frac{x^4 - x^2}{x^3 + 2x^2 + x}$$

$$\text{b. } \left(\frac{3x}{(x-2)^2} - \frac{3}{x-2} \right) : \frac{1}{x-2}$$

$$\text{c. } \frac{x-2}{x^2} + \frac{x+2}{x^2-x} - \frac{1}{x^2-1}$$

$$\text{f. } (5x^2 - 4x + 2) \cdot [(2x^2 - 3x + 2) - (2x + 1)(x^2 - 2x)]$$

$$\text{g. } (7x^2 - 3)^2 - (x + 2)(x - 2) - (2x + 1)(x - 2)$$

$$\text{h. } \frac{2x^2 + 2}{x^3 - x^2 + x - 1}$$

$$\text{i. } \frac{6x^2 - 6x - 12}{3x^3 - 12x^2 + 12x}$$

$$\text{d. } \frac{2x+2}{3x} : \frac{x+1}{6x^2}$$

$$\text{e. } \left(\frac{x+1}{x-1} - \frac{x-1}{x+1} \right) : \left(\frac{4x}{x^2-1} \right)$$

$$\text{j. } \frac{2x+1}{x^2-1} + \frac{x-1}{x+1} - \frac{1}{x-1}$$

$$\text{k. } \frac{2x^2-2x}{x+1} \cdot \frac{2x+2}{3x^2}$$

8. Calcula m para que el polinomio $P(x) = x^3 - mx^2 + 5x - 2$ sea divisible por $x + 1$

9. Desarrolla:

- $(3x^2 - 1)^2$
- $\left(\frac{1}{2}x + 2\right)\left(\frac{1}{2}x - 2\right)$
- $(x^2 - 2x + 3)^2$
- $(x - 2)^3$
- $(x - 2)^2 - (x + 2)^2 + (x - 2)(x + 2)$
- $(-2 + 3x)^2$
- $(-2 - 3x)^2$
- $(3x^3 - 1)(x^2 - 2x + 1)$
- $(x - \sqrt{2})(x + \sqrt{2})$
- $(x^2 - 2x + 3)^2$
- $\left(\frac{5}{3}x - 1\right)(4x - 2)^2 - (x^3 + x^2)$

10. Resuelve los sistemas:

- $$\left. \begin{array}{l} x + y \geq 4 \\ x \leq y \\ x \geq 0 \end{array} \right\}$$
- $$\left. \begin{array}{l} x + y \geq 4 \\ x \leq y \\ x \geq 0 \\ y \leq 6 \end{array} \right\}$$
- $$\left. \begin{array}{l} y \geq 2x - 1 \\ x \geq 0 \\ 0 \leq y \leq 3 \end{array} \right\}$$
- $$\left. \begin{array}{l} x + y \geq 30 \\ x \geq 5 \\ y \leq 10 \end{array} \right\}$$
- $$\left. \begin{array}{l} x + y \geq 30 \\ x \leq 5 \\ y \leq 10 \end{array} \right\}$$
- $$\left. \begin{array}{l} x - 2y \geq 1 \\ 0 \leq x \leq 3 \end{array} \right\}$$

11. Resuelve:

- $\frac{x+2}{x-3} < 0$
- $\frac{x+2}{x-3} \leq 0$
- $\frac{x+4}{2x-4} > 0$
- $\frac{2x-6}{x+3} \leq 0$
- $$\left. \begin{array}{l} x - 1 \leq 5 \\ 3x - 2 > 1 \end{array} \right\}$$
- $\frac{3x-2}{4} - \frac{2x+1}{3} > \frac{2(x+1)}{2}$
- $x^2 - x - 2 \geq 0$
- $-x^2 + 4x > 2x - 3$
- $$\left. \begin{array}{l} x^2 - x - 2 \geq 0 \\ -2x + 7 \geq \frac{x}{2} - 3 \end{array} \right\}$$

12. Saca factor común e identifica los productos notables:

- $2x^4 + 12x^3 + 18x^2 =$
- $20x^3 - 60x^2 + 45x =$
- $27x^3 - 3xy^2 =$
- $3x^3 + 6x^2y + 3y^2x =$
- $4x^4 - 81x^2y^2 =$
- $3x^3 - 12x =$
- $4x^3 - 24x^2 + 36x =$
- $45x^2 - 5x^4 =$
- $x^4 + x^2 + 2x^3 =$
- $x^6 - 16x^2 =$
- $16x^4 - 9 =$

13. Resuelve:

- $(3x - 2)(x + 4)(3x - 5) = 0$
- $x^3 - 4x^2 - 4x + 16 = 0$
- $x^4 - 3x^2 + 2 = 0$
- $x^4 - 3x^2 = 0$
- $\sqrt{4x + 1} - 2x + 5 = x + 2$
- $\sqrt{x^2 - 2x} + \sqrt{3x - 6} = 0$
- $\frac{2x}{x^2-1} - \frac{x+1}{x-1} = \frac{-5}{4}$
- $\frac{(x-2)^2}{3x+1} \cdot \frac{9x^2-1}{x-2} = 22$
- $2^{x^2-1} = 256$
- $3^{x+1} + 3^{x-2} = \frac{28}{3}$
- $3^x = 5$